Name: Answer key Per: _____ Date: _____

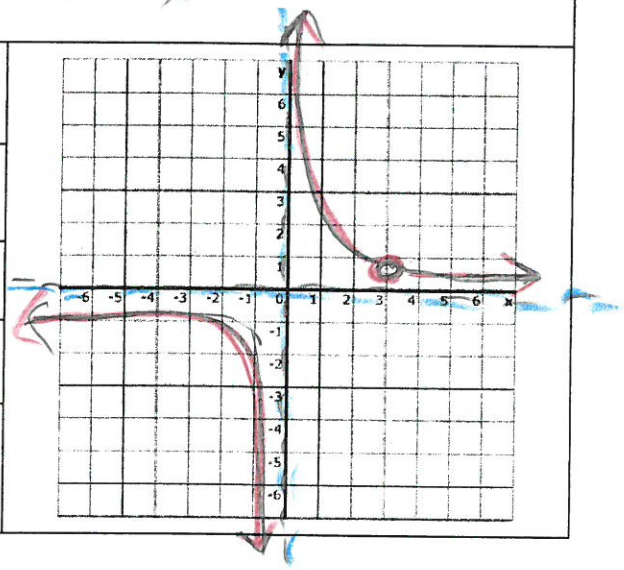
Serafino ▪ Precalculus S2

12B2 Rational Functions: Simplifying, Analyzing, Graphing

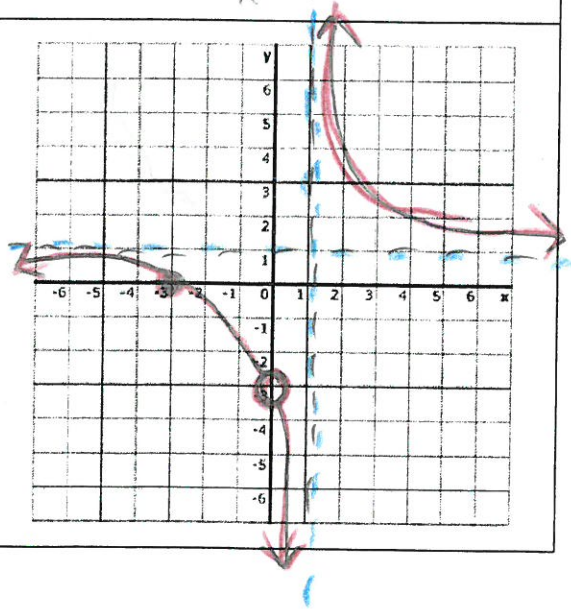
Do all necessary work on a separate paper and fill out the table and graph the function. *There is NOT enough room for work to go here.* You should be able to sketch all the functions with the information provided on the table. Use Desmos.com to check your work until the key is up.

★ Bonus: Look up a) when a rational function has a slant/oblique asymptote b) how to find the equation of it

1. $f(x)=\frac{2x-6}{x^2-3x}$	Factored & Simplified: $\frac{2(x-3)}{x(x-3)}$ $y=\frac{2}{x}$ $x\neq 3,0$
Domain Discontinuities: $x=0,3$	RDs (points): $(3,2/3)$
VA(s): $x=0$	HA: $y=0$
x–int(s): none	y–int: none
As $x\to-\infty$, $f(x)\to 0$	As $x\to\infty$, $f(x)\to 0$
Oblique (slant) Asymptote: none	



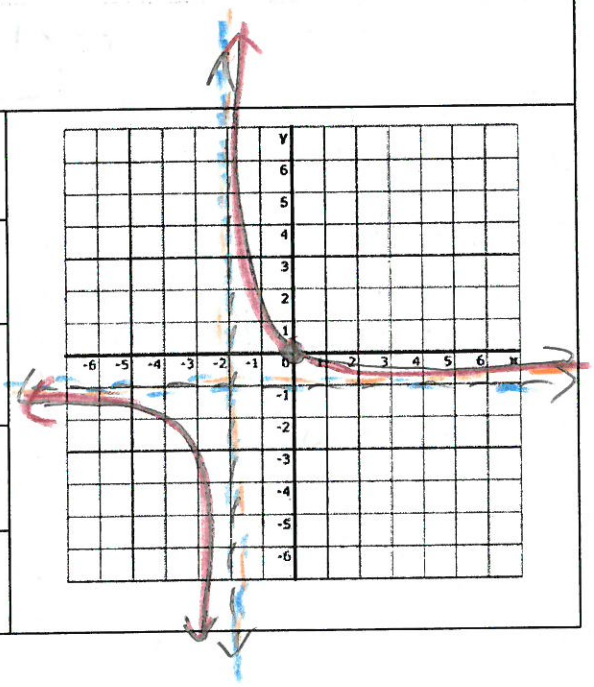
2. $f(x)=\frac{x^2+3x}{x^2-x}$	Factored & Simplified: $\frac{x(x+3)}{x(x-1)}$ $y=\frac{x+3}{x-1}$ $x\neq 0,1$
Domain Discontinuities: $x\neq 0,1$	RDs (points): $(0,-3)$
VA(s): $x=1$	HA: $y=1$
x–int(s): $(-3,0)$	y–int: none
As $x\to-\infty$, $f(x)\to 1$	As $x\to\infty$, $f(x)\to 1$
Oblique (slant) Asymptote: none	



3. $f(x) = \dfrac{x}{-x-2}$

Factored & Simplified: $y = \dfrac{x}{-(x+2)}$

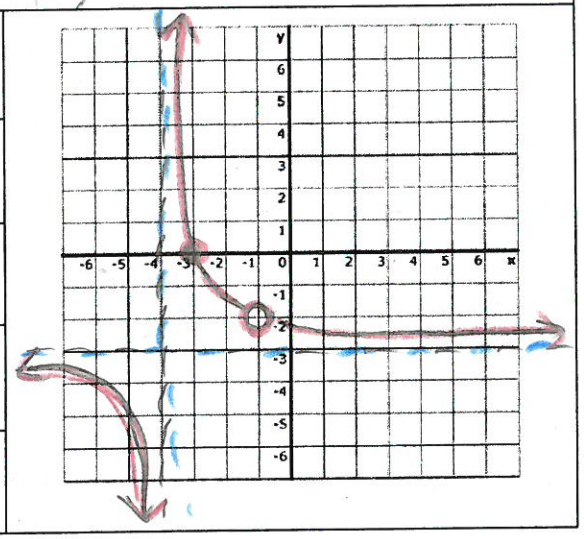
Domain Discontinuities: $x \neq -2$	RDs (points): none
VA(s): $x = -2$	HA: $y = -1$
x–int(s): $(0,0)$	y–int: $(0,0)$
As $x \to -\infty$, $f(x) \to -1$	As $x \to \infty$, $f(x) \to -1$
Oblique (slant) Asymptote: none	



4. $f(x) = \dfrac{-3x^2 - 12x - 9}{x^2 + 5x + 4}$

Factored & Simplified: $\dfrac{-3(x^2+4x+3)}{x^2+5x+4}$ $\dfrac{-3(x+1)(x+3)}{(x+1)(x+4)}$ $y = \dfrac{-3(x+3)}{x+4}$

Domain Discontinuities: $x \neq -1, -4$	RDs (points): $(-1, -2)$
VA(s): $x = -4$	HA: $y = -3$
x–int(s): $(-3,0)$	y–int: $(0, -9/4)$
As $x \to -\infty$, $f(x) \to -3$	As $x \to \infty$, $f(x) \to -3$
Oblique (slant) Asymptote: none	



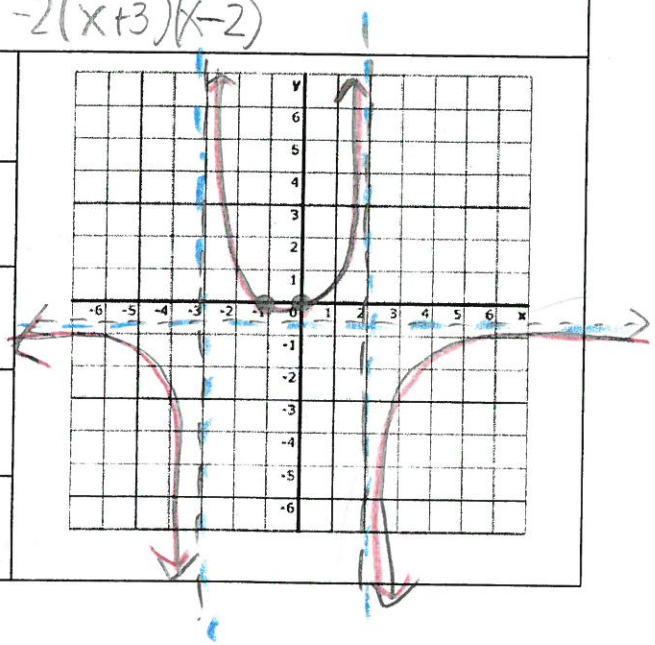
$f(-1) = \dfrac{-3(-1+3)}{-1+4} = \dfrac{-3(2)}{3}$

$f(0) = \dfrac{-9}{4}$

5. $f(x) = \dfrac{x^2 + x}{-2x^2 - 2x + 12}$

Factored & Simplified: $\dfrac{x(x+1)}{-2(x^2+x-6)}$ $y = \dfrac{x(x+1)}{-2(x+3)(x-2)}$

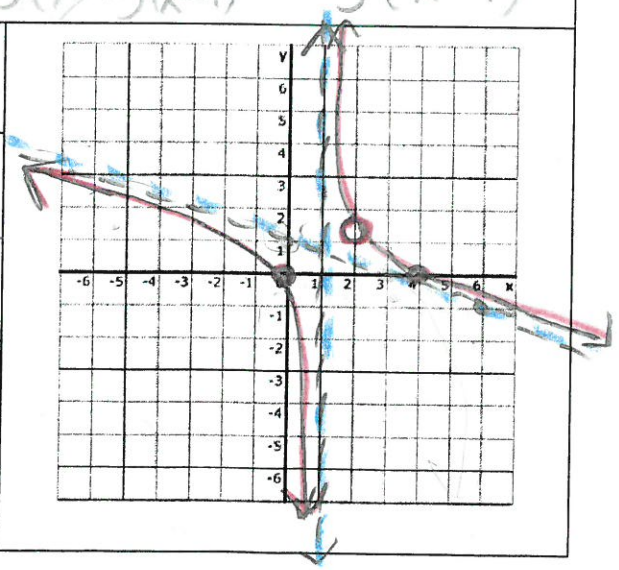
Domain Discontinuities: $x \neq -3, 2$	RDs (points): none
VA(s): $x = -3, x = 2$	HA: $y = -\frac{1}{2}$
x–int(s): $(0,0)$ $(-1,0)$	y–int: $(0,0)$
As $x \to -\infty$, $f(x) \to -\frac{1}{2}$	As $x \to \infty$, $f(x) \to -1/2$
Oblique (slant) Asymptote: none	



6. $f(x) = \dfrac{x^3 - 6x^2 + 8x}{-3x^2 + 9x - 6}$

Factored & Simplified: $\dfrac{x(x^2-6x+8)}{-3(x^2-3x+2)}$ $\dfrac{x(x-4)(x-2)}{-3(x-2)(x-1)}$ $y = \dfrac{x(x-4)}{-3(x-1)}$

Domain Discontinuities: $x \neq 2, 1$	RDs (points): $(2, 4/3)$
VA(s): $x = 1$	HA: none
x–int(s): $(0,0)$ $(4,0)$	y–int: $(0,0)$
As $x \to -\infty$, $f(x) \to \infty$	As $x \to \infty$, $f(x) \to -\infty$
Oblique (slant) Asymptote: $y = -\frac{1}{3}(x-3)$ or $y = -\frac{1}{3}x + 1$	

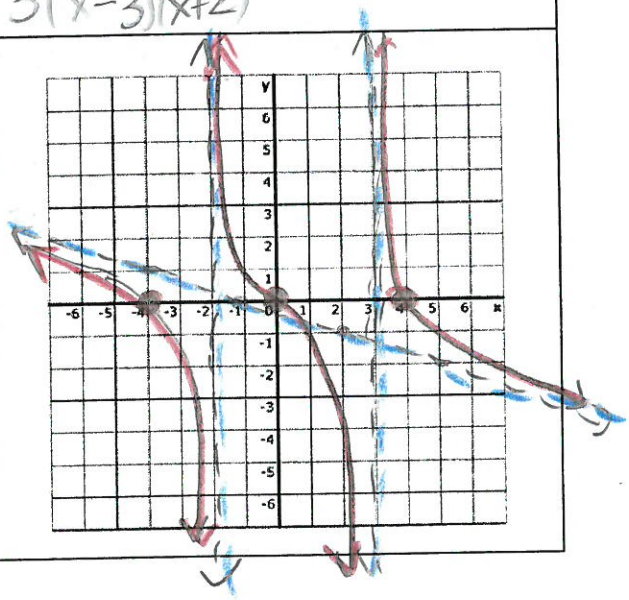


$\dfrac{2(2-4)}{-3(2-1)}$ $\dfrac{2(-2)}{-3}$

7. $f(x) = \dfrac{x^3 - 16x}{-3x^2 + 3x + 18}$

Factored & Simplified: $\dfrac{x(x^2-16)}{-3(x^2-x-6)}$ $y = \dfrac{x(x-4)(x+4)}{-3(x-3)(x+2)}$

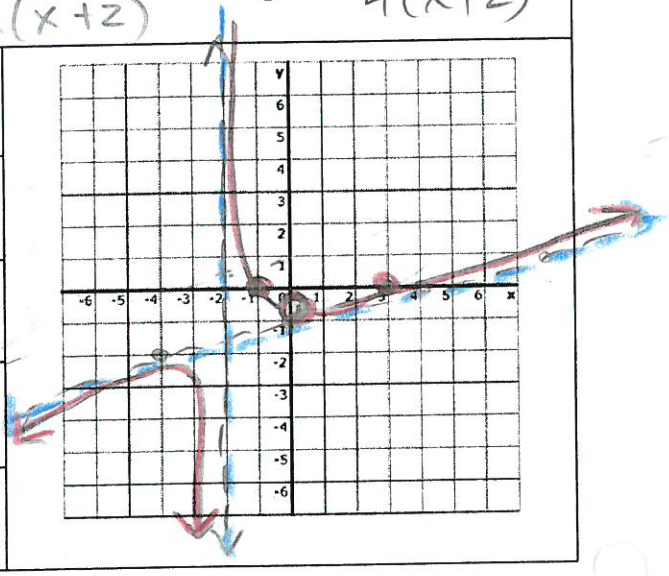
Domain Discontinuities: $x \neq 3, -2$	RDs (points): none
VA(s): $x = 3, x = 2$	HA: none
x–int(s): $(0,0)$ $(4,0)$ $(-4,0)$	y–int: $(0,0)$
As $x \to -\infty$, $f(x) \to \infty$	As $x \to \infty$, $f(x) \to -\infty$
Oblique (slant) Asymptote: $y = -\frac{1}{3}(x+1)$ or $y = -\frac{1}{3}x - \frac{1}{3}$	



8. $f(x) = \dfrac{x^3 - 2x^2 - 3x}{4x^2 + 8x}$

Factored & Simplified: $\dfrac{x(x^2-2x-3)}{4x(x+2)}$ $\dfrac{x(x-3)(x+1)}{4x(x+2)}$ $y = \dfrac{(x-3)(x+1)}{4(x+2)}$

Domain Discontinuities: $x \neq 0, -2$	RDs (points): $(0, -3/8)$
VA(s): $x = -2$	HA: none
x–int(s): $(3,0)$ $(-1,0)$	y–int: none
As $x \to -\infty$, $f(x) \to -\infty$	As $x \to \infty$, $f(x) \to \infty$
Oblique (slant) Asymptote: $y = \frac{1}{4}(x+4)$ or $y = \frac{1}{4}x + 1$	

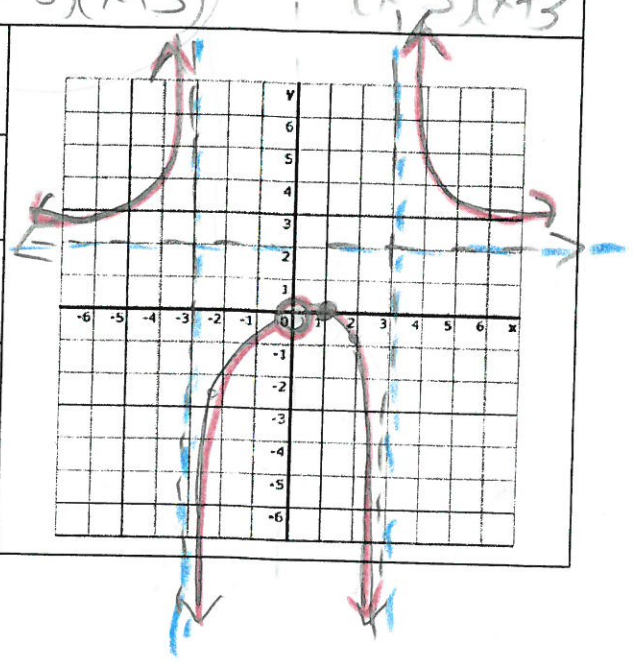


✓ cancel!

9. $f(x)=\dfrac{2x^3-2x^2}{x^3-9x}$

Factored & Simplified: $\dfrac{2x^2(x-1)}{x(x^2-9)}$ $\dfrac{2x^2(x-1)}{x(x-3)(x+3)}$ $y=\dfrac{2x(x-1)}{(x-3)(x+3)}$

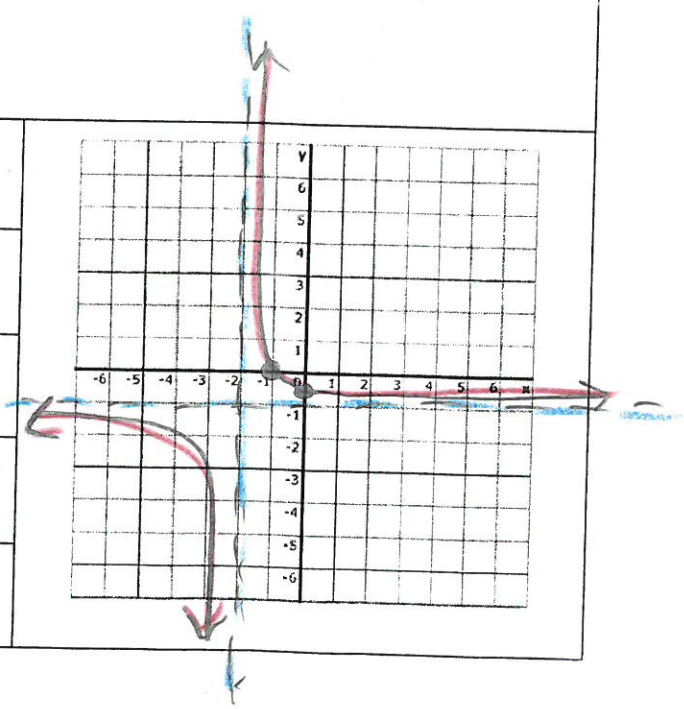
Domain Discontinuities: $x\neq 0,-3,3$	RDs (points): $(0,0)$
VA(s): $x=3, x=-3$	HA: $y=2$
x–int(s): $(1,0)$	y–int: none
As $x\to-\infty$, $f(x)\to 2$	As $x\to\infty$, $f(x)\to 2$
Oblique (slant) Asymptote: none	



10. $f(x)=\dfrac{-x-1}{x+2}$ $y=\dfrac{-(x+1)}{x+2}$

Factored & Simplified:

Domain Discontinuities: $x\neq -2$	RDs (points): none
VA(s): $x=-2$	HA: $y=-1$
x–int(s): $(-1,0)$	y–int: $(0,-1/2)$
As $x\to-\infty$, $f(x)\to -1$	As $x\to\infty$, $f(x)\to -1$
Oblique (slant) Asymptote: none	



11. $f(x) = \dfrac{x^3 - x}{x^3 + 2x^2 - 3x}$

Factored & Simplified: $\dfrac{x(x^2-1)}{x(x^2+2x-3)} \quad \dfrac{x(x-1)(x+1)}{x(x+3)(x-1)} \quad y = \dfrac{x+1}{x+3}$

Domain Discontinuities: $x \neq 0, -3, 1$

RDs (points): $(0, 1/3), (1, 1/2)$

VA(s): $x = -3$

HA: $y = 1$

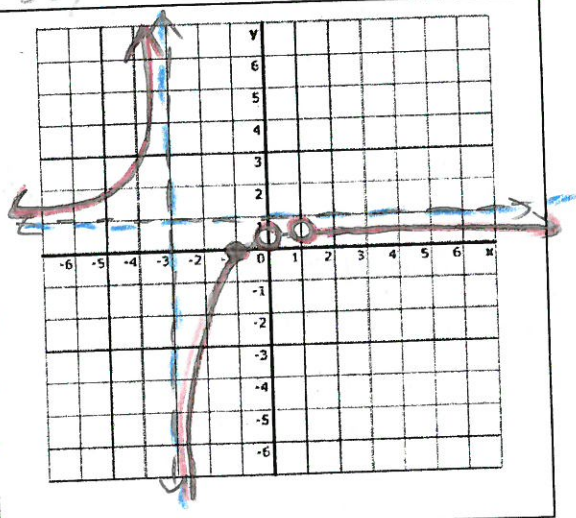
x–int(s): $(-1, 0)$

y–int: none

As $x \to -\infty$, $f(x) \to 1$

As $x \to \infty$, $f(x) \to 1$

Oblique (slant) Asymptote: none



$-\frac{1}{3}x \ldots$

12. $f(x) = \dfrac{x^2 + 3x + 2}{-3x - 12}$

Factored & Simplified: $\dfrac{(x+1)(x+2)}{-3(x+4)}$

Domain Discontinuities: $x \neq -4$

RDs (points): none

VA(s): $x = -4$

HA: none

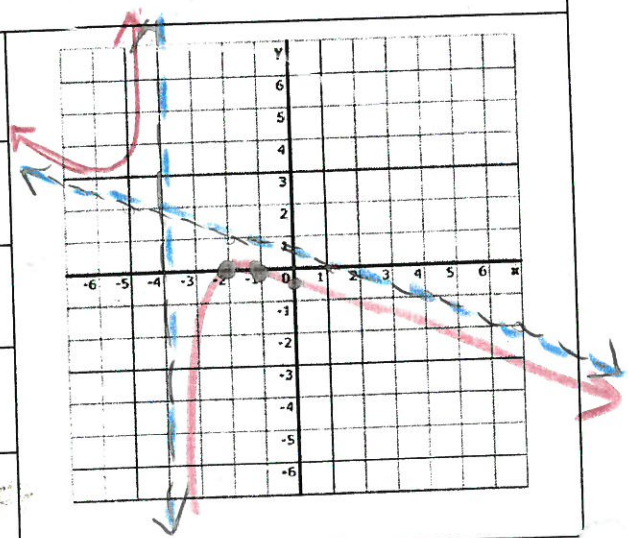
x–int(s): $(-1, 0) \quad (-2, 0)$

y–int: $(0, -1/6)$

As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to -\infty$

Oblique (slant) Asymptote: $y = -\frac{1}{3}(x-1)$ or $y = -\frac{1}{3}x + \frac{1}{3}$



$f(0) = \dfrac{(1)(2)}{-3(4)_2} =$